

## Appendix A

### A. Gold Sequence Generator

Due to the large range of values of the cross-correlations of the maximum-length shift-register sequence, sometimes called  $m$ -sequence, they usually are not adopted to CDMA-like applications. PN-sequences with better periodic cross-correlation properties than  $m$ -sequences are given by the sequences of Gold (1967, 1968) and Kasami (1966), where the two-values of their sequences are  $+1$  and  $-1$ . In Robert Gold, "Maximal recursive sequences with 3-valued recursive cross-correlation functions," *IEEE Trans. on Inform. Theory*, vol. 41, pp. 154-156, January 1968 [21], the contents of which are hereby expressly incorporated by reference, it was proved that certain pairs of  $m$ -sequences of length  $N$  exhibit a three-valued cross-correlation function [21] with the values  $-1, -t(m), t(m)-2$ , where

$$t(m) = \begin{cases} 2^{(m+2)/2} + 1 & (\text{odd } m) \\ 2^{(m+2)/2} - 1 & (\text{even } m) \end{cases}$$

and where  $m$  denotes an  $m$ -stage shift register of sequence length  $N = 2^m - 1$ .

Consider the generation of the Gold sequences of length  $N = 31 = 2^5 - 1$  with the pair of preferred sequences, that are obtained from the book by Peterson and Weldon (W. W. Peterson and E. J. Weldon, *Error Correcting Codes*, MIT Press, Cambridge, MA, 1972 [22], the contents of which are hereby expressly incorporated by reference) are described by the polynomials,

$$\begin{aligned} g_1(D) &= D^5 + D^2 + 1, \\ g_2(D) &= D^5 + D^4 + D^3 + D^2 + 1 \end{aligned}$$

Let  $b$  and  $b'$  represent the two  $m$ -sequences with period  $N = 31$  that are generated by  $g_1(D)$  and  $g_2(D)$ , respectively. The Gold family for the sequences of length 31 comprises of  $2^m + 1 = 33$  sequences given in R. L. Peterson, R. E. Ziemer, and D. E. Borth, *Introduction to Spread Spectrum Communications*, Prentice Hall, Englewood Cliffs, NJ, 1995 [23], the contents of which are hereby expressly incorporated by reference, by

$$G = \{b\} \cup \{b'\} \cup \{ \{b + D^\tau b'\} \mid 0 \leq \tau \leq N-1 \}, \quad (56)$$

5 where the term  $D^\tau b'$  represents a phase shift of the  $m$ -sequence  $b'$  by  $\tau$  units and “ $\cup$ ” denotes set union. The sequences  $s_1$  and  $s_2$  in the example of Section 2 are derived by setting  $\tau$  in Eq. (56) equal to 0 and 1, respectively.